

Binomial Distribution

Binomial Distribution is the probability of a SUCCESS or FAILURE outcome in an experiment that is repeated multiple times.

Binomial is a type of distribution that has TWO possible outcomes "bi-"

A basketball player's shot percentage would found to be 70%. In other words, this player made 70% of the shots taken. Find the probability of scoring EXACTLY 2 shots out of 6 attempts.

let X represent the # of shots made
We want to find $P(X=2)$

S S m m m m

$$0.7 \cdot 0.7 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = (0.7)^2 (0.3)^4$$

BUT WAIT...

It could also be like this

m S m S m m

$$0.3 \cdot 0.7 \cdot 0.3 \cdot 0.7 \cdot 0.3 \cdot 0.3 = (0.7)^2 (0.3)^4$$

How many ways can we place two items out of six? $\Rightarrow \binom{6}{2} = {}_2C_6$

$$P(X=2) = \binom{6}{2} \cdot (0.7)^2 (0.3)^4 = 0.59 = 59\%$$

$$B(n, k) = \binom{n}{k} p^k (1-p)^{n-k}$$

n - # of trials
 k - # of successes desired
 p - probability of getting a success in one trial
 $1-p$ - probability of getting a failure in one trial

Find the probability of flipping a coin 10 times and getting EXACTLY 6 heads.

$$\left. \begin{array}{l} n = 10 \\ k = 6 \\ p = 0.5 \\ 1-p = 0.5 \end{array} \right\} \Rightarrow \binom{10}{6} (0.5)^6 (0.5)^4 = 20\%$$

A fair die is thrown four times. Calculate the probabilities of getting:

a. 0 Twos

$$P(X=0) = \binom{4}{0} \left(\frac{1}{6}\right)^0 \left(\frac{5}{6}\right)^4 = 48\%$$

b. 1 two

$$P(X=1) = \binom{4}{1} \left(\frac{1}{6}\right)^1 \left(\frac{5}{6}\right)^3 = 38\%$$

c. 2 twos

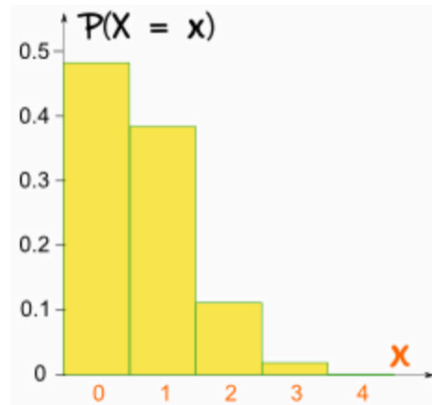
$$P(X=2) = \binom{4}{2} \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 = 11\%$$

d. 3 twos

$$P(X=3) = \binom{4}{3} \left(\frac{1}{6}\right)^3 \left(\frac{5}{6}\right)^1 = 1.5\%$$

e. 4 twos

$$P(X=4) = \binom{4}{4} \left(\frac{1}{6}\right)^4 \left(\frac{5}{6}\right)^0 = 0.08\%$$



Criteria

- ① Fixed # of trials
- ② Each trial is Independent of the others
- ③ There are only two outcomes
- ④ The Probability of each outcome remains constant from trial to trial

A packet of 10 vegetable seeds has a germination rate of 96%. What is the probability that:

a. At least 9 seeds will sprout

$$\begin{aligned}P(X=9 \cup X=10) &= P(X=9) + P(X=10) \\&= \binom{10}{9}(0.96)^9(0.04)^1 + \binom{10}{10}(0.96)^{10}(0.04)^0 \\&= 0.25 + 0.66 \\&= 91\%\end{aligned}$$

b. At most 8 seeds will sprout

$$1 - P(X=9 \cup X=10) = 1 - 0.91 = 0.09 = 9\%$$